

PVC Calendering: A Simplified Prediction Technique*

A. RAY, *Chemistry Department, Indian Institute of Technology, New Delhi 110 016, India*, and A. V. SHENOY, *Polymer Science and Engineering Group, Chemical Engineering Division, National Chemical Laboratory, Pune 411 008, India*

Synopsis

A simplified technique for prediction of pressure distribution, velocity profiles, torque, and power requirements for the calendering of power-law fluids is presented. The technique assumes isothermal conditions, is based on the lubrication approximation and Gaskell's analysis, and uses melt flow index as the only temperature-dependent resin parameter for the prediction. The predictions are found to compare favorably with those made by Vlachopoulos and Hrymak for rigid PVC.

INTRODUCTION

The term "calendering" as applied to thermoplastic materials refers to the shaping of the material into sheet or film by feeding it through a pair of corotating heated rolls. This process is commonly used for the manufacture of various poly(vinyl chloride) (PVC) products such as leather cloth, shrink films for packaging, resilient flooring tiles, etc. During the manufacturing process, the molten PVC formulations are subjected to a shearing flow at shear rates around $10\text{--}10^2\text{ s}^{-1}$ within a temperature range $150\text{--}220^\circ\text{C}$. As the viscosity of the molten PVC mass is dependent on both shear rate and temperature, the processor ought to possess a thorough idea of the melt rheology of the PVC formulation of interest. However, more often than not, the practical processor finds answers to processing problems related to rheology through simple thumbrules based on experience, and rightly so, because of the highly theoretical approaches of rheologists which lead only to complicated equations and complex computer solutions. Truly speaking, the works of rheologists and the practicing processor ought not to be compartmentalized as they are a single community with interdependence.

The present article is intended to provide a simplistic approach for getting quick estimates of various critical parameters during PVC calendering. An idea about the pressure distribution on the rollers, the torque exerted by each roll, the power input into each roll and the average temperature rise due to viscous dissipation is provided through an easily measurable rheological parameter. In this sense, the present communication is an attempt

* NCL Communication No. 3334.

at bridging the gap between the theoretical rheologist and the practical processor.

BACKGROUND

The "calendering" process for shaping of thermoplastic materials into sheets and films has been the subject of investigation for over three decades now. Gaskell¹ was the first to analyse the process by developing a mathematical procedure in one dimension for Newtonian fluids, derived as a specific extension for a purely viscous (nonelastic and time-independent) fluid. McKelvey² extended the analysis to include non-Newtonian flow, assuming only that the viscosity depends on the state of shear and making the corresponding modification in the momentum equation. He also showed the approximate cubic dependence of maximum pressure developed between rolls on the leave-off distance, and specialized his non-Newtonian development for power-law fluids.

Alston and Astill³ considered yet another special case—that of fluids whose shear rate dependent viscosity could be approximated by a hyperbolic tangent function. Tadmor and Gogos⁴ pointed out the limitations of an isothermal treatment and set up equations assuming the lubrication approximation for a CEF model that exhibited normal stresses in viscometric flows. Agassant and Avenas⁵ developed a thermopseudoplastic model using Brazinsky's model⁶ to obtain force and torque as functions of plasticity index and process parameters, assuming conformity of thermodependence of the behavior with experimental data. It should be noted that the Agassant-Avenas model, which rightly takes into account the heating of the melt by the viscous dissipation and therefore attempts a simultaneous solution of the mechanical and energy equations, is cumbersome to use in calculation since it requires iterations based on linear relationships between inlet and outlet data.

In a recent paper, Vlachopoulos and Hrymak⁷ have suggested a hydrodynamic model for the calendering of rigid PVC, using both the isothermal assumption and the lubrication approximation. It is the purpose of this paper to present a similar model in which the pressure distribution is dependent only on one easily-measurable parameter apart from its usual dependence on process constants so that quick estimates of the critical design parameters in PVC calendering can be obtained.

THEORETICAL ANALYSIS

The Hydrodynamic Theory without Slip

The present form of the hydrodynamic theory makes essentially the same assumption for the operating conditions as Gaskell¹ and Vlachopoulos and Hrymak⁷, namely, isothermal steady-state flow of an incompressible power-law liquid. The reconciliation of this assumption to the experimental data is also identical; namely, we assume that the calendering temperature is equal to the average surface temperature plus the average temperature rise due to adiabatic viscous dissipation, as per Kiparissides and Vlachopoulos.⁸

For the system shown in Figure 1, the lubrication assumption, as applied

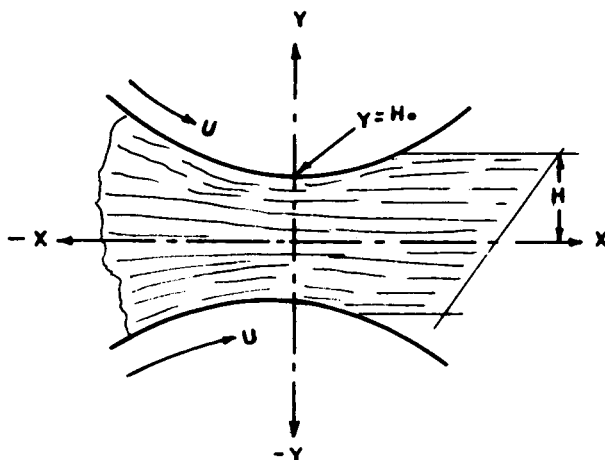


Fig. 1. The nip area and notation for a pair of corotating and equal-sized rolls which have no translational motion of their own.

to the mass flow balance and momentum conservation (details of which are available in Middleman⁹) yield

$$Q = 2 \int_0^h u \, dy \quad (1)$$

$$\frac{\partial P}{\partial x} = \frac{\partial}{\partial y} (\tau_{xy}) \quad (2)$$

At this stage, it is essential to substitute an expression for the shear stress based on an appropriate rheological model. A number of rheological models have been proposed to describe the viscosity behavior of polymer melts, solutions, and suspensions.¹⁰ Models that have good theoretical foundations¹¹ are known to lack the flexibility to fit a wide range of non-Newtonian materials whereas some empirical models have been found to correlate viscosity data adequately well through a limited number of meaningful parameters. In the high shear rate region, the empirical model which is most commonly used is the Ostwald-de Waele power-law model¹⁰ of the following form:

$$\tau_{xy} = m \left(\frac{du}{dy} \right)^n \quad (3)$$

This particular form has been used by Vlachopoulos and Hrymak⁷ for curve-fitting the rheological data for molten PVC. The m and n values were dependent on temperature,⁷ and hence a series of expressions had to be generated in order to evaluate process parameters in calendering at temperatures of interest. The values of m and n which are basically obtained from the rheogram would change even with changes in the PVC formulations. Thus, using an expression of the kind shown in eq. (3) for derivation

of critical parameters in calendering would mean that the knowledge of the complete flow curve constructed from data obtained on expensive sophisticated rheological equipments becomes essential. Collection of such information is undoubtedly beyond the financial means and technical capabilities of the processor; and it is in such circumstances that the processor resorts to the usual rule of thumb approach. What is truly required is a balance between the accuracy at high costs and the order of magnitude estimations through simplified procedures in order to upgrade the thumb rule approach. With this in view, Shenoy et al.¹² have provided a procedure for coalescing the rheograms of different types of PVC formulations at various temperatures into a unique master curve. From the coalesced curve shown in Figure 2 as reproduced from Shenoy et al.¹² the rheograms of any formulation at the temperature of interest can be easily estimated from the easily determinable flow parameter, namely, the melt flow index (MFI). The MFI value can be measured readily on the inexpensive melt flow indexer in accordance with ASTM test conditions D3364-73. In order to eliminate the need for reading out values from the coalesced curve during the process of generating the rheograms, a modified Ostwald-de Waele power-law model of the type given in eq. (3) is fitted in the shear rate range of $10\text{--}10^3 \text{ s}^{-1}$ as

$$\eta \cdot \text{MFI} = K(\dot{\gamma}/\text{MFI})^{n-1} \quad (4)$$

or, in other words,

$$\tau_{xy} = \frac{K}{(\text{MFI})^n} \left(\frac{du}{dy} \right)^n \quad (5)$$

where K and n are temperature-invariant and also independent of the type of formulation. In the case of PVC, the determined values of K and n are the following:

$$K = 4.6 \times 10^5 \text{ (g/cm s}^{2-n} \text{) (g/10 min)}^n \quad n = 0.36 \quad (6)$$

Equation (5) in lieu of eq. (3) would thus provide a far more convenient predictive approach for the calendering process. Thus substitution of eq. (5) in eq. (2) and assuming a no-slip condition, i.e., $u(h) = U$ at the roll surface, the following can be written:

$$\frac{\partial P}{\partial x^*} = -\frac{K}{(\text{MFI})^n} \left(\frac{2n+1}{n} \right)^n \left(\frac{U}{H_0} \right)^n \left(\frac{2R}{H_0} \right)^{1/2} \left[\frac{(\lambda^2 - x^{*2})\lambda^2 - x^{*2n-1}}{(1 + x^{*2})^{2n+1}} \right] \quad (7)$$

where

$$x^* = x(2RH_0)^{-1/2}, \text{ a dimensionless distance} \quad (8)$$

and

$$\lambda = (Q/2UH_0 - 1)^{1/2}, \text{ a dimensionless flow parameter} \quad (9)$$

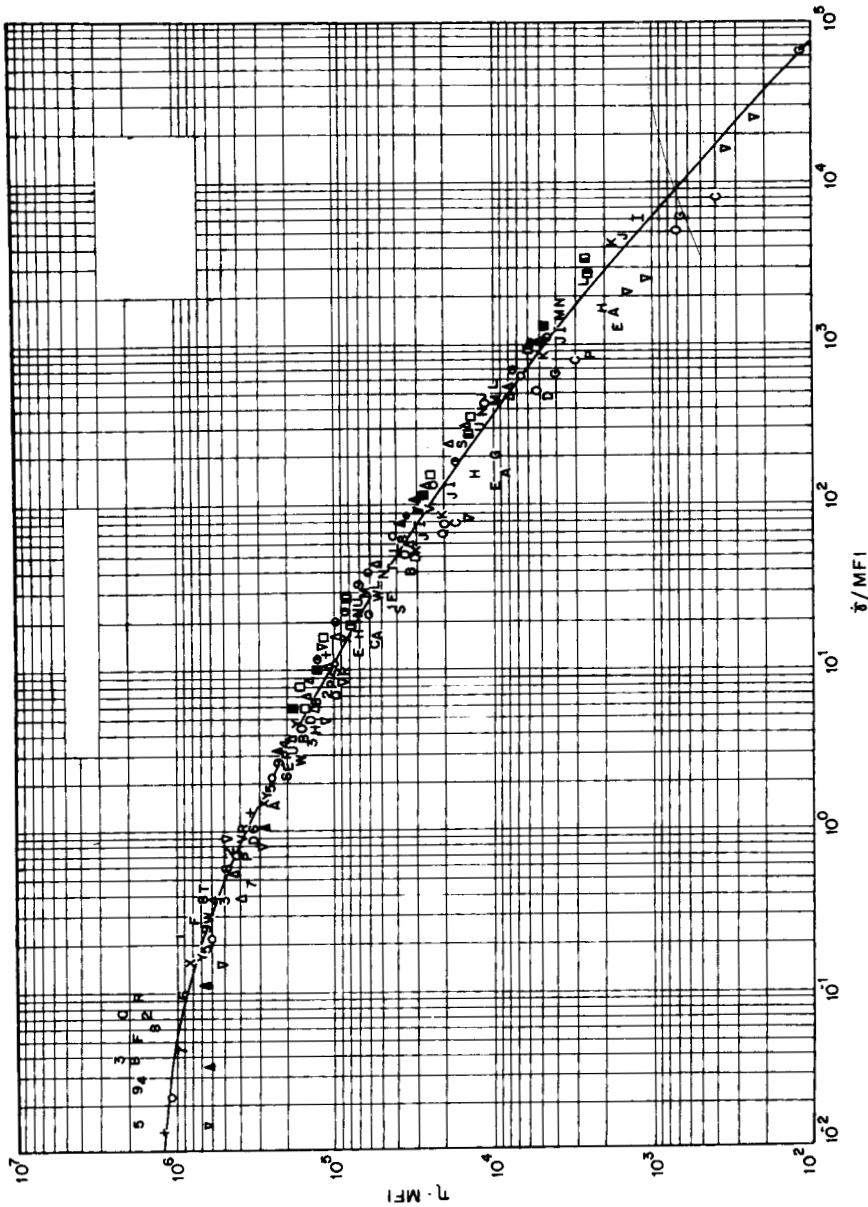


Fig. 2. Master curve for viscosity versus shear rate for PVC formulations as taken from Ref. 12. The solid line shows the best fit line conferring to eq. (4): η is in poise, $\dot{\gamma}$ (s^{-1}), MFI ($g/10$ min).

Assuming an infinite reservoir for the PVC melt feed, we have the boundary condition

$$x^* \rightarrow -\infty \Rightarrow P \rightarrow 0 \quad (10)$$

$$x^* = \lambda \Rightarrow P = 0 \quad (11)$$

Integration of (7) using eq. (10) yields

$$P(\text{MFI})^n = C_1 C_2 \int_{x^*}^{\lambda} \frac{|\lambda^2 - x^{*2}|^{n-1} (\lambda^2 - x^{*2}) dx^*}{(1 + x^{*2})^{2n+1}} \quad (12)$$

where

$$C_1 = K \left(\frac{2n+1}{n} \right)^n \quad (13)$$

$$C_2 = \left(\frac{U}{H_0} \right)^n \left(\frac{2R}{H_0} \right)^{1/2} \quad (14)$$

Note that C_1 is a constant for a given resin and C_2 for given process parameters. In the case of PVC, it follows from eq. (6) that

$$C_1 = 8.0775 \times 10^5 (\text{g/cm s}^{2-n}) (\text{g/10 min})^n$$

From the condition given in eq. (10), we have an expression for determining λ as follows:

$$\int_{-\infty}^{\lambda} \frac{|\lambda^2 - x^{*2}|^{n-1} (\lambda^2 - x^{*2}) dx^*}{(1 + x^{*2})^{2n+1}} = 0 \quad (15)$$

A plot of λ as a function of n has been made by Vlachopoulos and Hrymak,⁷ and this can be used for the specific case of $n = 0.36$ given in eq. (6) to yield $\lambda = 0.503$. Using this value of λ , eq. (12) can be numerically integrated for various values of x^* to yield a unique plot of $P(\text{MFI})^n$ vs. x^* for given process parameters in the case of all PVC formulations as shown in Figure 3. In the present case, the numerical integration was carried out using the trapezoidal rule. Taking an appropriate value of MFI for the specific PVC formulation under consideration at the operating temperature, curves of P vs. x^* can be generated from this master plot.

The Hydrodynamic Theory with Slip

Any analysis of flow behavior in calendering is incomplete without a consideration of the corrections introduced due to slip—which could arise, for instance, by sliding of high molecular weight material over a thin layer of low molecular weight comprising of oligomeric material and lubricants, etc., which tends to adhere to the wall. As per a power-law expression given by Chauffoureaux et al.,¹³ approximating such slip, we can write

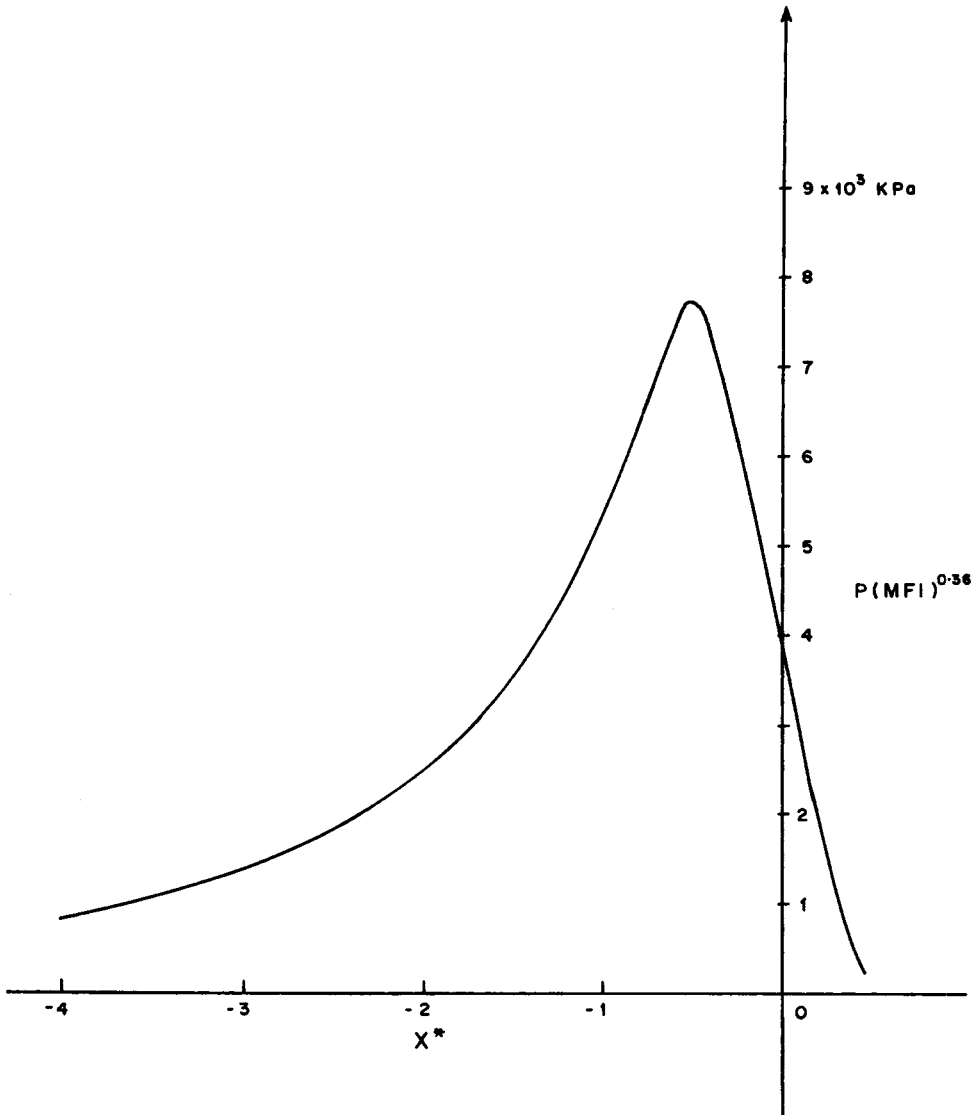


Fig. 3. Plot of $P(MFI)^{0.36}$ vs. dimensionless distance x^* — master curve for the theory without slip.

$$u = U - U_0 - \frac{1}{\beta} r_w^\alpha \quad (16)$$

Using this as a boundary condition at $y = h$ along with the constraint that

$$\frac{\partial u}{\partial y} = 0 \text{ at } y = 0 \quad (17)$$

integration of eq. (5) yields

$$u = (\text{MFI}) \left(\frac{n}{n+1} \right) \left(\frac{1}{K} \right)^{1/n} \left| \frac{\partial P}{\partial x} \right| \left| \frac{\partial P}{\partial x} \right|^{(1-n/n)} (y^{(n+1/n)} - h^{(n+1/n)}) \\ + U - U_0 - \frac{1}{\beta} \left(\frac{\partial P}{\partial x} \right) \left| \frac{\partial P}{\partial x} \right|^{\alpha-1} h^\alpha \quad (18)$$

Integration of the above expression to obtain the volumetric flow rate from the equation

$$Q = 2 \int_0^h u \, dy \quad (19)$$

yields

$$\frac{Q}{2h} = U - U_0 - \frac{1}{\beta} \left(\frac{\partial P}{\partial x} \right) \left| \frac{\partial P}{\partial x} \right|^{\alpha-1} h^\alpha - (\text{MFI})^{n/(n+1)} \\ \left(\frac{1}{K} \right)^{1/n} \left(\frac{\partial P}{\partial x} \right) \left| \frac{\partial P}{\partial x} \right|^{(1-n)/n} h^{(n+1)/n} \quad (20)$$

Introducing

$$x^* = x(2RH_0)^{-1/2} \quad (21)$$

and

$$\lambda = \left[\frac{Q}{2(U - U_0)H_0} - 1 \right]^{1/2} \quad (22)$$

where $h = H_0(1 + x^{*2})$ from geometry as shown by Middleman,⁹ we obtain

$$\left(\frac{\partial P}{\partial x^*} \right) \left\{ B(x^*) \left| \frac{\partial P}{\partial x^*} \right|^{\alpha-1} + C(x^*) \left| \frac{\partial P}{\partial x^*} \right|^{(1-n)/n} \right\} = A(x^*) \quad (23)$$

where

$$A(x^*) = (U - U_0) (1 + x^{*2})^{-1} (\lambda^2 - x^{*2}) \quad (24)$$

$$B(x^*) = \frac{1}{\beta} H_0^\alpha (1 + x^{*2})^\alpha (2RH_0)^{-\alpha/2} \quad (25)$$

$$C(x^*) = \left(\frac{n}{2n+1} \right) (\text{MFI}) \left(\frac{1}{K} \right)^{1/n} (2RH_0)^{-1/2n} H_0^{(n+1)/n} (1 + x^{*2})^{(n+1)/n} \quad (26)$$

The equation can only be solved by a tedious numerical technique which involves calculation of $\partial P / \partial x^*$ values for a number of x^* positions by a

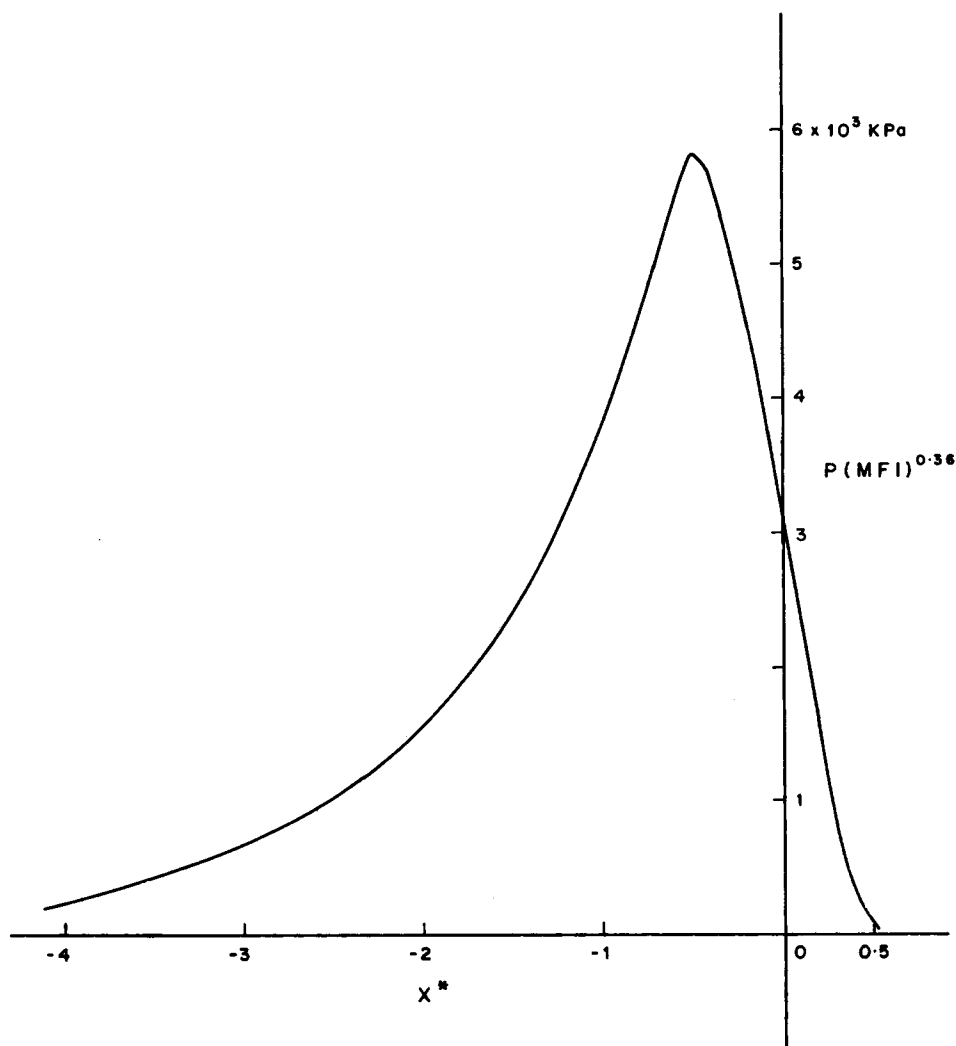


Fig. 4. Master curve as in Figure 3 for the theory with the effect of slip taken into account.

bisection linear interpolation technique as given in McCormick and Salvadori.¹⁴ These values are inserted into a fourth-order Runge-Kutta method to obtain the P vs. x^* data.

If we make a simplifying assumption that the inaccuracies introduced by neglecting slip beyond a first-order estimate are small, we can determine a master curve for P vs. x^* (taking $\text{MFI} = 1$) by the numerical analysis suggested above and use this to compute P vs. x^* for other temperatures using the equation from the no slip analysis, namely,

$$P_1(\text{MFI}_1)^n = P_2(\text{MFI}_2)^n \quad \text{at constant } x^* \quad (27)$$

It is found that this approximation introduces an error of the order of -10% in P , which is well within the limits of the errors introduced by measurement

TABLE I
Relevant Rheological Parameters

Temp. (°C)	Vlachopoulos-Hrymak ^{7 a}		Present work ^b (MFI)
	m (Pa s ⁿ)	n	
160	7.81×10^4	0.456	0.454
170	5.26×10^4	0.390	1.154
180	5.06×10^4	0.322	1.43
182	4.61×10^4	0.340	1.80
190	2.82×10^4	0.412	5.10
200	1.38×10^4	0.479	17.3
210	0.59×10^4	0.583	42.3

^a $\tau = m\dot{\gamma}^n$

^b Equation for shear stress versus shear rate, $\tau = K(\dot{\gamma}/\text{MFI})^n$, where K and n are given by eq. (6).

uncertainties in other pressure prediction method as, for example, in Vlachopoulos and Hrymak.⁷ The master curve for the case involving the conditions of slip is shown in Figure 4.

RESULTS AND DISCUSSION

A comparison of the predictions of the present approach with that suggested by Vlachopoulos and Hrymak⁷ is carried out to demonstrate the propriety of the technique. The rheological data on the PVC formulations at various temperatures and the relevant process parameters have been taken from the above reference⁷ in order to provide a meaningful comparison. The MFI values at each temperature were calculated from the τ vs. $\dot{\gamma}$ relationship provided by Vlachopoulos and Hrymak,⁷ using the methodology suggested by Shenoy et al.¹² Table I lists the rheological parameters used while the process constants are given in Table II. Using these values, the resulting pressure distribution as a function of x were derived from the master plot given in Figure 3 for various temperatures and compared with Vlachopoulos and Hrymak's predictions for the case without slip. The plots have been shown in Figures 5(a)–5(f). The predictions of maximum pressure from the present method are 10–25% lower than those of Vlachopoulos and Hrymak⁷ for temperatures of 170–210°C. However, at 160°C, the values of P_{\max} are markedly different. It is observed that while Vlachopoulos and Hrymak⁷ predict a P_{\max} of 19.8×10^3 kPa at 160°C and a value of 9.7×10^3 kPa at 170°C, the present technique suggests corresponding values of

TABLE II
Relevant Process Parameters

Parameter	Value
U	0.0738 ms ⁻¹
U_0	0 ms ⁻¹
H_0	3×10^{-4} m
R	0.125 m
β	2.644×10^{14} m ⁻¹ s ⁻¹ Pa ^{-a}
α	2.28

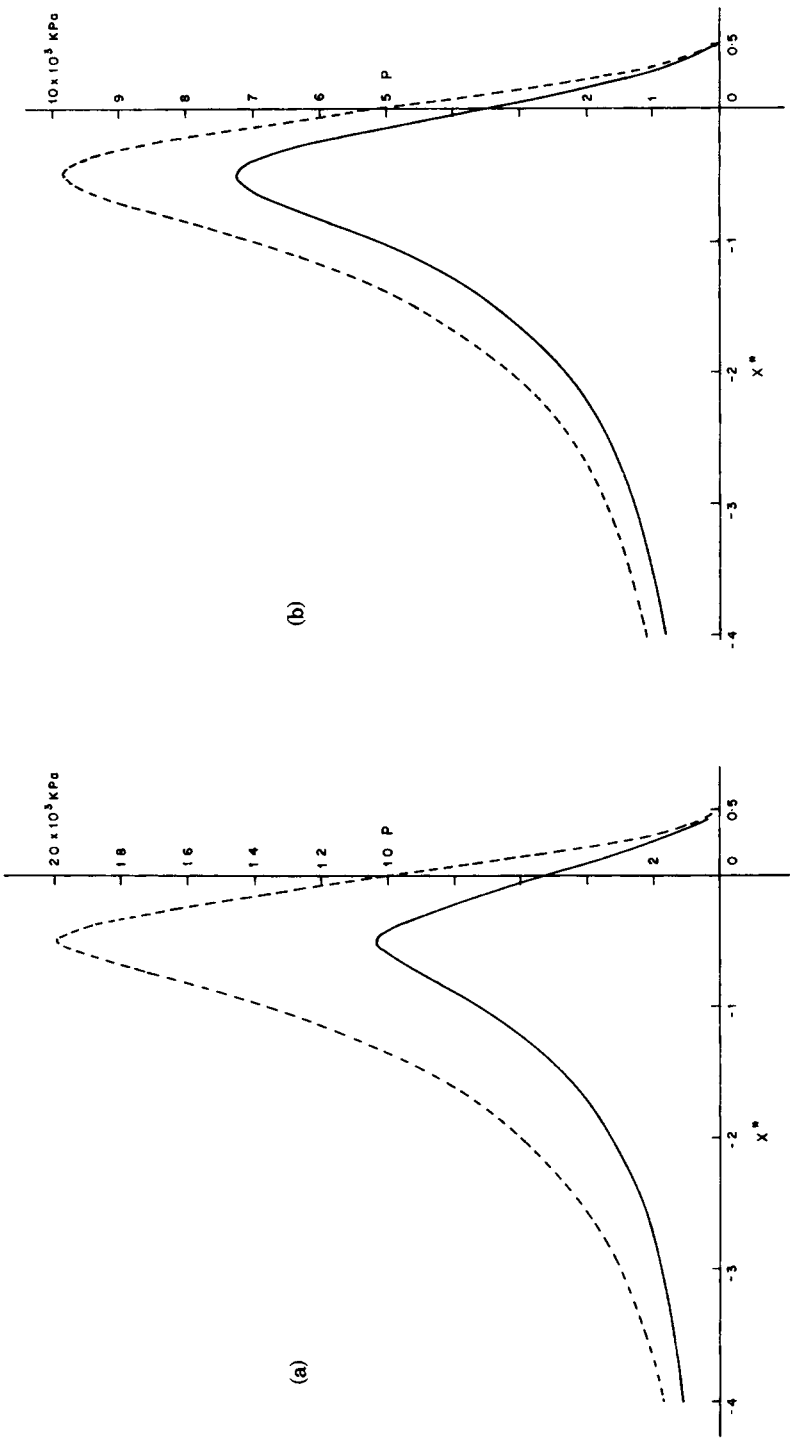
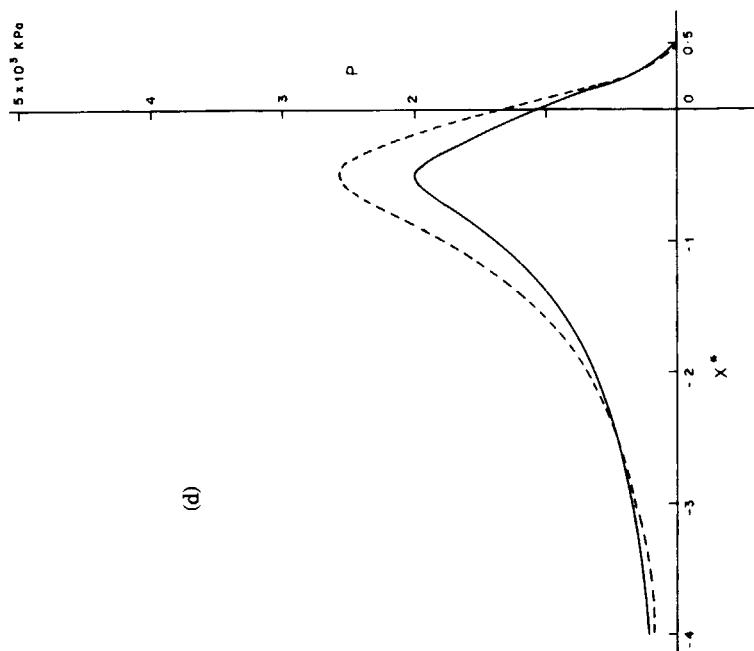
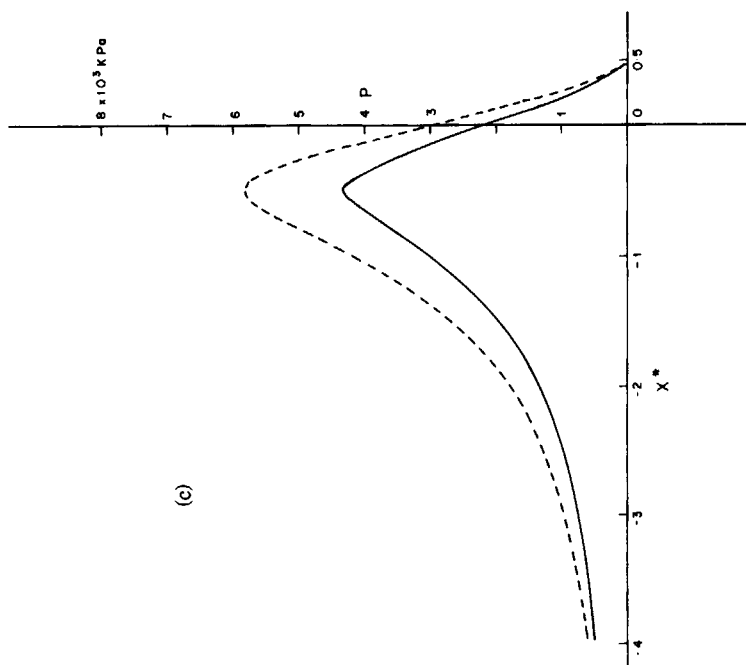


Fig. 5(a)-(f). Comparisons of predictions by Vlachopoulos and Hrymak's analysis⁷ (---) and the present work (—), showing pressure profiles at temperatures varying from 160°C to 210°C at 10°C intervals.



(d)



(c)

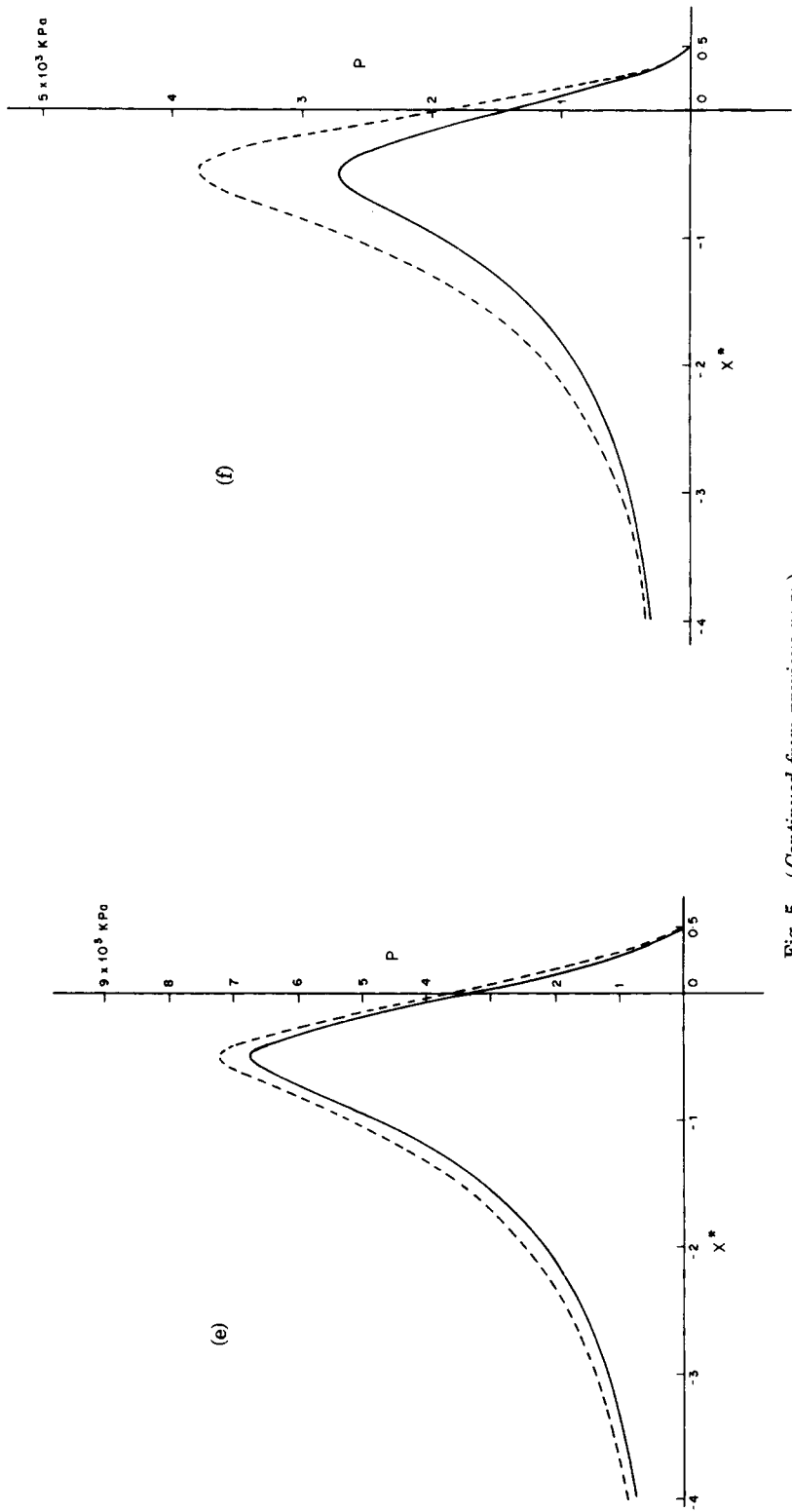


Fig. 5. (Continued from previous page.)

10.3×10^3 kPa and 7.5×10^3 kPa, respectively. It is not very likely that such a phenomenal pressure change as derivable from Ref. 7 will occur over only a 10°C temperature change. The present approach thus predicts more realistic values. Further, it can be seen that the prenip slope of the pressure distribution changes more gradually in the present analysis which also is more realistic. A comparison of the two sets of predictions along with the experimental data used in Ref. 7 is shown in Figure 6. It can be seen that the present approach provides a closer fit to experimental data. A similar observation is made in the "with slip" case, as can be seen from Figure 7. It is worth mentioning that for this figure an exact analysis of eq. (23) was carried out using the appropriate MFI value to yield $P_{\max} = 5.2 \times 10^3$ kPa. However, if the master curve (Fig. 4) is used, the predicted

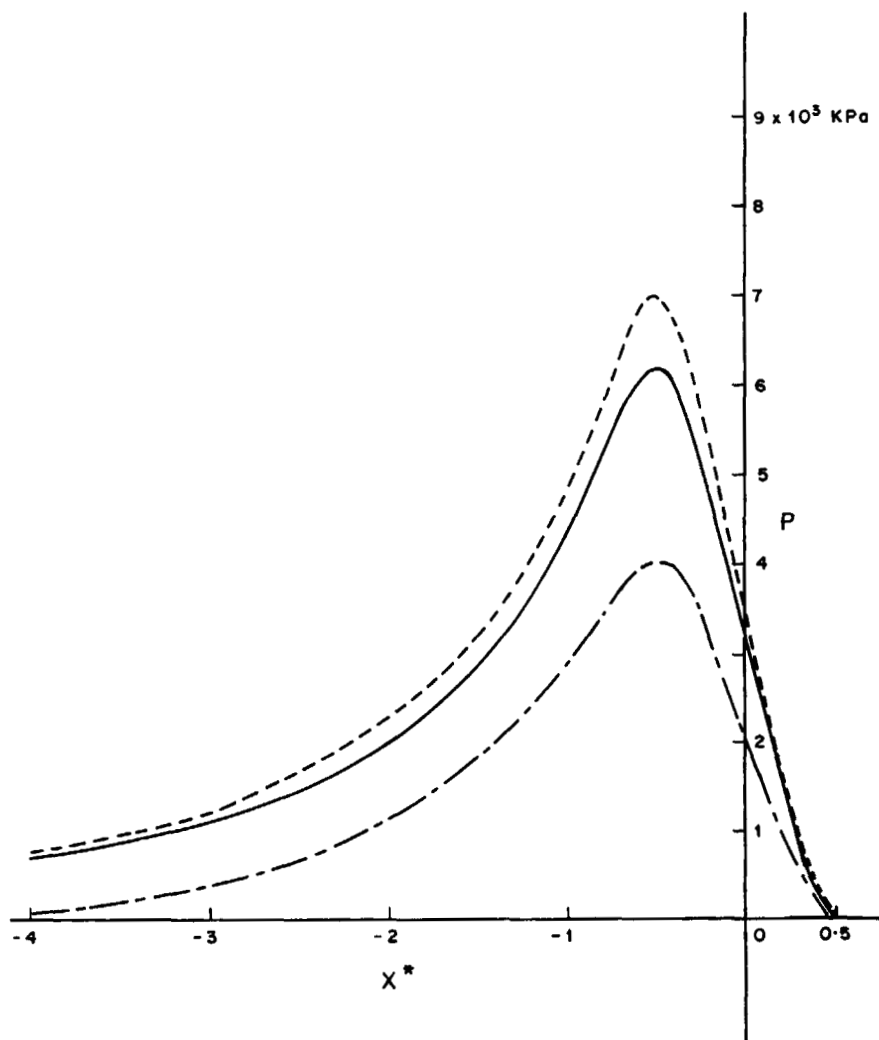


Fig. 6. Comparison of predictions made in Ref. 7 (---), those made in the present work (—), and an experimental curve⁷ (- · -) for the theory without slip.

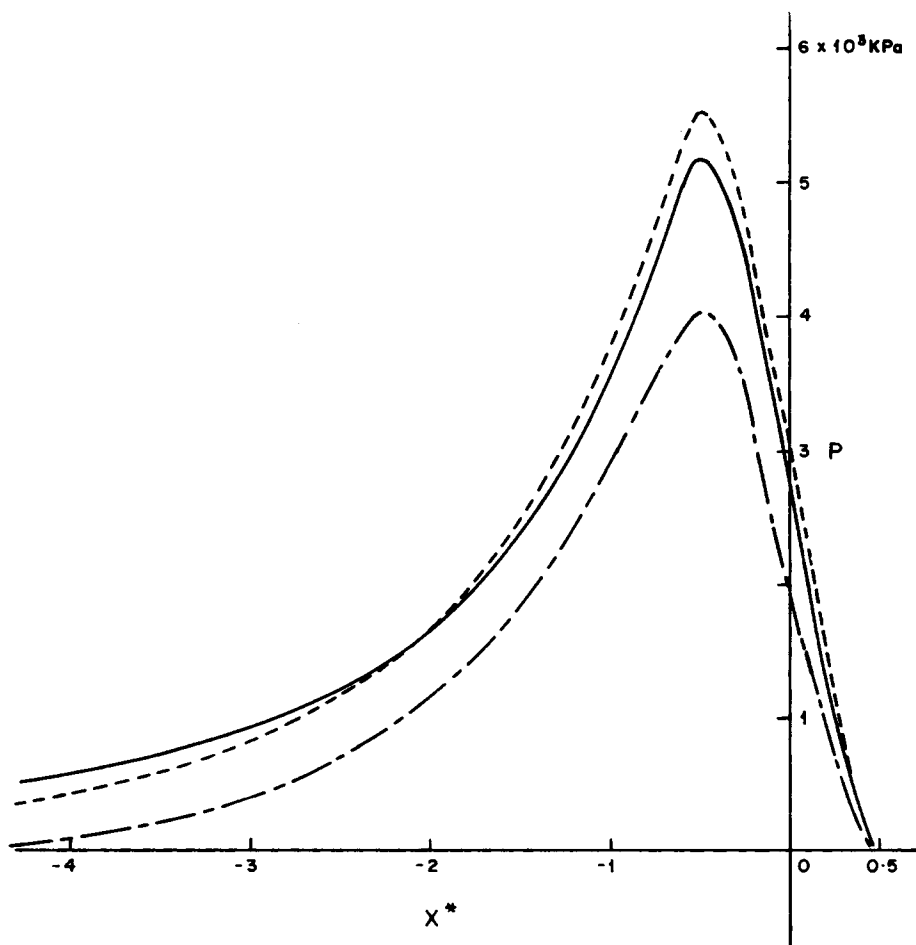


Fig. 7. Comparisons as in Figure 6 for the theory with slip.

value of P_{\max} is slightly lower and yields a value of $P_{\max} = 4.8 \times 10^3$ kPa, which compares favorably with the experimental value of 4.2×10^3 kPa.

It must be noted that the sensitivity of the dependence of P on λ , which itself depends on n , is quite significant. Thus the method suggested by Vlachopoulos and Hrymak⁷ is prone to uncertainties for each measurement on n , i.e., each temperature of use. In the present paper, the value of n is invariant, and the only temperature-dependent variable in the pressure prediction equation is the melt flow index, which can be measured repeatedly for accuracy without resorting to curve-fitting techniques and data correction procedures, as in the case of Ref. 7.

The sheet thickness has been shown by Middleman⁹ to be approximately equal to

$$H = H_0(1 + \lambda^2) \quad (28)$$

As λ is a constant for this technique, it is unable to predict sheet thickness.

However, Vlachopoulos and Hrymak's plots⁷ show that sheet thickness dependence on n is such that H/H_0 does not change appreciably in usual working ranges of PVC formulation.

Proceeding along the lines of Vlachopoulos and Hrymak,⁷ other process parameters of interest can be estimated as follows: The torque exerted by each roll is obtained by integration of the product of shear stress at the surface, roll radius, and contact area between the melt and roll, namely,

$$\Gamma = \int \tau_w R dA \quad (29)$$

where

$$A = W dx \quad (30)$$

$$\tau_w = h \left(\frac{\partial P}{\partial x} \right) \quad (31)$$

Making substitutions for x and h in terms of x^* and H_0 , we obtain

$$\Gamma = WRH_0 \int_{-x\delta}^{\lambda} \left(\frac{\partial P}{\partial x^*} \right) (1 + x^{*2}) dx^* \quad (32)$$

Applying eq. (7), the above equation can be rearranged to

$$\Gamma(\text{MFI})^n = C_1(WRH_0C_2) \int_{-x\delta}^{\lambda} \frac{|\lambda^2 - x^{*2}|^{n-1} (\lambda^2 - x^{*2})}{(1 + x^{*2})^{2n}} dx^* \quad (33)$$

Once again, the expression yields a constant curve (as λ and n are constants) for $\Gamma(\text{MFI})^n$ vs. x^* . Similarly, the power input into each roll would be given by

$$\phi = \int \tau_w \cdot \mathbf{U} \cdot dA \quad (34)$$

Again, by appropriate substitutions,

$$\phi(\text{MFI})^n = C_1(WUH_0)C_2 \int_{-x\delta}^{\lambda} \frac{|\lambda^2 - x^{*2}|^{n-1} (\lambda^2 - x^{*2})}{(1 + x^{*2})^{2n}} dx^* \quad (35)$$

The average temperature rise as a consequences of viscous dissipation⁷ can be estimated by dividing total power input to the two relevant rolls by the mass flow rate and the specific heat of the melt, i.e.,

$$(\Delta T)_{\text{avg}} = \frac{\phi_A + \phi_B}{\dot{m}C_p} \quad (36)$$

This can also be seen to be a function of MFI and C_p alone for fixed

TABLE III
Comparison between Predicted Torque and Power Requirements by Vlachopoulos-Hrymak⁷ and Present Work

Roll no.	Temp (°C)	Vlachopoulos-Hrymak ⁷		Present work	
		Torque (J)	Power (W)	Torque (J)	Power (W)
3	172	253	150	202	120
4	178	213	126	194	114

process conditions, and since the variation in C_p is small — 1.84 J/gK at 150°C to 1.80 J/gK at 200°C,⁷ the variation in MFI alone will adequately provide a reasonable estimate of $(\Delta T)_{\text{avg}}$ at different process temperatures.

Vlachopoulos and Hrymak⁷ have used appropriate values of m and n at 172°C and 178°C in order to predict the torque exerted on rolls nos. 3 and 4 of their experimental setup. In the present case, using simply the MFI value estimated at the temperature of consideration, the torque value can be derived from eq. (33). Similarly, the power requirements can be found using eq. (35) and compared with values predicted through the Vlachopoulos and Hrymak⁷ approach. Table III shows the comparison. It can be seen that the values predicted by the proposed technique are in reasonable agreement with the predictions of Vlachopoulos and Hrymak,⁷ considering the simplicity of the present approach over that of the earlier work.⁷

CONCLUSION

A modification of the isothermal model for PVC calendering⁷ is suggested. The nip region pressure distribution predicted by this model is in good agreement with experimental data and with the previous analysis.⁷ Although several assumptions, e.g., isothermal conditions and the lubrication approximation have been made, the model is relatively more convenient to use as compared to its predecessor⁷ as it formulates pressure as the function of only *one* resin parameter, namely, the melt flow index, which can be conveniently measured using elementary equipment.

NOMENCLATURE

C_p	specific heat
C_1	defined in eq. (13)
C_2	defined in eq. (14)
h	distance from center plane to roll periphery
H	half-width of gap at exit equal to half-thickness of sheet produced
H_0	one-half of minimum gap width
K	consistency index (constant)
m	consistency index (variable) i.e. apparent viscosity at shear rate $\dot{\gamma} = 1 \text{ s}^{-1}$
\dot{m}	mass flow rate
MFI	melt flow index (g/10 min)
n	power law index
P	pressure
Q	volumetric flow rate
R	roll radius
T	temperature
u	velocity

U_0	parameter in slip boundary condition
W	sheet width
x	coordinate in the direction of flow
x^*	dimensionless distance, defined in eq. (8)
y	coordinate normal to the direction of flow
<i>Greek letters</i>	
α, β	parameters in the slip boundary condition
$\dot{\gamma}$	shear rate (s^{-1})
Γ	torque
$(\Delta T)_{avg}$	average temperature rise
λ	dimensionless "leave-off" distance, defined in eq. (9)
ϕ	power
τ	shear stress
τ_{xy}	shear stress
τ_w	shear stress at roll surface

References

1. R. E. Gaskell, *J. Appl. Mech.*, **17**, 334 (1950).
2. J. M. McKelvey, *Polymer Processing*, Wiley, New York, 1962.
3. W. W. Alston and K. N. Astill, *J. Appl. Polym. Sci.*, **17**, 3157 (1973).
4. Z. Tadmor and C. G. Gogos, *Principles of Polymer Processing*, Wiley, New York, 1979.
5. J. F. Agassant and P. Avenas, *J. Macromol. Sci. Phys.*, **14**, 345 (1977).
6. I. Brazinsky, H. F. Cosway, C. F. Valle, Jr., R. K. Jones, and V. Story, *J. Appl. Polym. Sci.*, **14**, 2771 (1970).
7. J. Vlachopoulos and A. N. Hrymak, *Polym. Eng. Sci.*, **20**, 725 (1980).
8. C. Kiparissides and J. Vlachopoulos, *Polym. Eng. Sci.*, **18**, 210 (1978).
9. S. Middleman, *Fundamentals of Polymer Processing*, McGraw-Hill, New York, 1977.
10. R. B. Bird, R. C. Armstrong, and O. Hassager, *Dynamics of Polymeric Liquids*, Vol. 1, *Fluid Mechanics*, Wiley, New York, 1977.
11. W. W. Graessley, *J. Chem. Phys.*, **54**, 5143 (1971).
12. A. V. Shenoy, D. R. Saini, and V. M. Nadkarni, *J. Vinyl Tech.*, **5**, 192 (1983).
13. J. C. Chauffoureaux, C. Dehennan, and J. van Rijckevorsel, *J. Rheol.*, **21**, 1 (1979).
14. J. M. McCormick and M. G. Salvadori, *Numerical Methods in FORTRAN*, Prentice-Hall, Englewood Cliffs, NJ, 1964.

Received December 7, 1983

Accepted September 20, 1984